M31

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Scéim Mharcála	*Scrúduithe Ardteistiméireachta, 2006*
Matamaitic Fheidhmeach	*Gnáthleibhéal*
Marking Scheme	*Leaving Certificate Examination, 2006*
Applied Mathematics	*Ordinary Level*

General Guidelines

1 Penalties of three types are applied to candidates' work as follows:

Slips	- numerical slips	S(-1)
Blunders	- mathematical errors	B(-3)
Misreading	- if not serious	M(-1)

Serious blunder or omission or misreading which oversimplifies:
- award the attempt mark only.

Attempt marks are awarded as follows: 5 (att 2), 10 (att 3).

2 Mark all answers, including excess answers and repeated answers whether cancelled or not, and award the marks for the best answers.

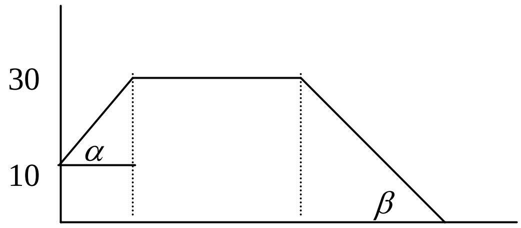
3 Mark scripts in red unless candidate uses red. If a candidate uses red, mark the script in blue or black.

4 Number the grid on each script 1 to 9 in numerical order, not the order of answering.

5 Scrutinise **all** pages of the answer book.

6 The marking scheme shows one correct solution to each question. In many cases there are other equally valid methods.

1. A car travels along a straight level road.
It passes a point p at a speed of 10 m/s and accelerates uniformly for 5 seconds to a speed of 30 m/s.
It then moves at a constant speed of 30 m/s for 9 seconds.
Finally the car decelerates uniformly from 30 m/s to rest at point q in 6 seconds.
Find **(i)** the acceleration
(ii) the deceleration
(iii) $|pq|$, the distance from p to q
(iv) the average speed of the car as it travels from p to q.



(i)

$v = u + at$ or $a = \tan\alpha$

$30 = 10 + 5a$ or $a = \dfrac{20}{5}$

$a = 4 \text{ m/s}^2$ $a = 4 \text{ m/s}^2$ 10

(ii)

$v = u + at$ or $a = \tan\beta$

$0 = 30 + 6a$ or $a = \dfrac{30}{6}$

$a = -5$ $a = 5$

deceleration is 5 m/s^2 10

(iii)

distance $= 5(10) + \frac{1}{2}(5)(20)$ 5

$+ (9)(30)$ 5

$+ \frac{1}{2}(6)(30)$ 5

$= 100 + 270 + 90$

$= 460 \text{ m}$ 5

(iv) average speed $= \dfrac{\text{total distance}}{\text{total time}}$

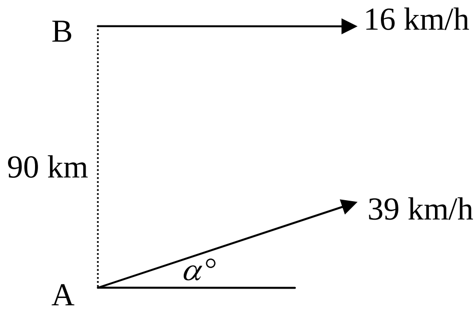
$= \dfrac{460}{20}$

$= 23 \text{ m/s}$ 10 | 50

2. Ship A is travelling east $\alpha°$ north with a constant speed of 39 km/h, where $\tan\alpha = \frac{5}{12}$.

Ship B is travelling due east with a constant speed of 16 km/h.

At 2 pm ship B is positioned 90 km due north of ship A.



(i) Express the velocity of ship A and the velocity of ship B in terms of $\vec{i}$ and $\vec{j}$.

(ii) Find the velocity of ship A relative to ship B in terms of $\vec{i}$ and $\vec{j}$.

(iii) Find the shortest distance between the ships.

(i)

$$\vec{V}_A = 39\cos\alpha\,\vec{i} + 39\sin\alpha\,\vec{j}$$ 5

$$= 39\left(\frac{12}{13}\right)\vec{i} + 39\left(\frac{5}{13}\right)\vec{j}$$

$$= 36\,\vec{i} + 15\,\vec{j}$$ 5

$$\vec{V}_B = 16\,\vec{i} + 0\,\vec{j}$$ 10

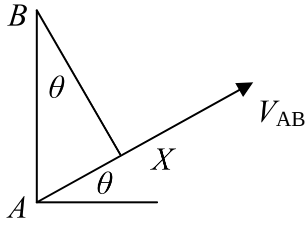
(ii)

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$ 5

$$= \left(36\,\vec{i} + 15\,\vec{j}\right) - \left(16\vec{i}\right)$$ 5,5

$$= 20\,\vec{i} + 15\,\vec{j}$$ 5

(iii)



$$\text{shortest distance} = |BX|$$

$$= 90\cos\theta$$ 5

$$= 90\left(\frac{20}{25}\right)$$

$$= 72 \text{ km}$$ 5

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3. A particle is projected from a point on a level horizontal plane with initial velocity $10\vec{i} + 35\vec{j}$ m/s, where $\vec{i}$ and $\vec{j}$ are unit perpendicular vectors in the horizontal and vertical directions respectively.

Find **(i)** the time it takes to reach the maximum height

(ii) the maximum height

(iii) the two times when the particle is at a height of 50 m

(iv) the speed with which the particle strikes the plane.

(i)

$$v_y = 0 \qquad\qquad v = u + at$$
$$35 - 10t = 0 \qquad\qquad 0 = 35 - 10t$$
$$t = 3.5 \text{ s} \qquad\qquad t = 3.5 \text{ s}$$

10

(ii)

$$\text{maximum ht.} = 35t + \tfrac{1}{2}at^2$$
$$= 35(3.5) - 5(3.5)^2$$
$$= 61.25 \text{ m}$$

10

(iii)

$$35t - 5t^2 = 50$$
$$t^2 - 7t + 10 = 0$$
$$(t-2)(t-5) = 0$$
$$t = 2\text{ s} \quad \text{and} \quad t = 5\text{ s}$$

20

(iv)

$$\text{time} = 7 \text{ seconds}$$
$$\text{velocity} = 10\vec{i} + (35-70)\vec{j}$$
$$= 10\vec{i} - 35\vec{j}$$

$$\text{speed} = \sqrt{10^2 + 35^2}$$
$$= 36.4 \text{ m/s}$$

10

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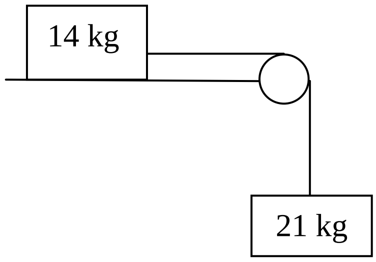
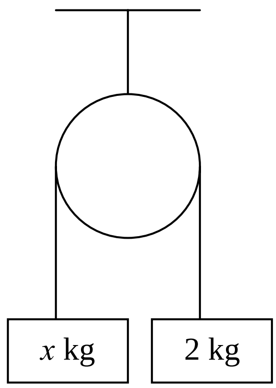
4. **(a)** Two particles of masses 14 kg and 21 kg are connected by a light, taut, inextensible string passing over a smooth light pulley at the edge of a rough horizontal table.
The coefficient of friction between the 14 kg mass and the table is $\frac{1}{2}$.
The system is released from rest.



(i) Show on separate diagrams the forces acting on each particle.
(ii) Find the common acceleration of the particles.

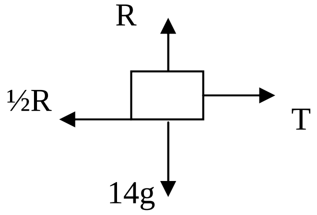
(b) A light inelastic string passes over a smooth light pulley.
A mass of x kg is attached to one end of the string and a mass of 2 kg is attached to the other end.

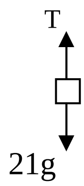


When the system is released from rest the 2 kg mass falls 3 metres in $\sqrt{6}$ seconds.

Find **(i)** the common acceleration
(ii) the tension in the string
(iii) the value of x.

(a) (i)





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(ii)

$T - \frac{1}{2}R = 14a$ 5

$R = 14g$ 5

$21g - T = 21a$ 5

$a = \frac{140}{35} = 4 \text{ m/s}^2$ 5

(b)

(i) $s = ut + \frac{1}{2}at^2$

$3 = 0 + \frac{1}{2}a(6)$

$a = 1 \text{ m/s}^2$ 5

(ii) $2g - T = 2a$

$20 - T = 2$

$T = 18$ N 10

(iii) $T - xg = xa$

$18 - 10x = x$

$x = \frac{18}{11}$ kg 5

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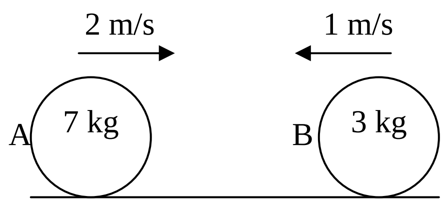
5. A smooth sphere A, of mass 7 kg, collides directly with another smooth sphere B, of mass 3 kg, on a smooth horizontal table.
A and B are moving in opposite directions with speeds of 2 m/s and 1 m/s respectively.
The coefficient of restitution for the collision is $\frac{1}{3}$.



Find **(i)** the speed of A and the speed of B after the collision

(ii) the loss in kinetic energy due to the collision

(iii) the magnitude of the impulse imparted to A due to the collision.

(i) PCM

$$7(2) + 3(-1) = 7v_1 + 3(v_2)$$
$$11 = 7v_1 + 3v_2 \qquad 10$$

NEL

$$v_1 - v_2 = -e(u_1 - u_2)$$
$$= -\frac{1}{3}(2+1)$$
$$= -1 \qquad 10$$

$$v_1 = 0.8\ m/s \text{ and } v_2 = 1.8 \text{ m/s} \qquad 10$$

(ii)

$$\text{KE before collision} = \tfrac{1}{2}(7)(2)^2 + \tfrac{1}{2}(3)(-1)^2$$
$$= 15.5 \qquad 5$$
$$\text{KE after collision} = \tfrac{1}{2}(7)(0.8)^2 + \tfrac{1}{2}(3)(1.8)^2$$
$$= 7.1 \qquad 5$$
$$\text{KE lost} = 15.5 - 7.1$$
$$= 8.4 \text{ J} \qquad 5$$

(iii)

$$\text{Impulse} = (7)(2) - (7)(0.8)$$
$$= 8.4 \text{ Ns} \qquad 5$$

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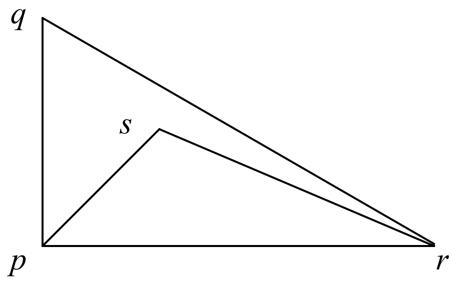
6. **(a)** Particles of weight 3 N, 7 N, 10 N and 15 N are placed at the points $(-4,-5)$, $(2,1)$, (x, y) and $(-1,3)$, respectively.
The centre of gravity of the four particles is at the origin.

Find the value of x and the value of y.

(b) A triangular lamina with vertices p, q and r has the triangular portion with vertices p, s and r removed.

The co-ordinates of the vertices are $p(0,0)$, $q(0,6)$, $r(12,0)$ and $s(3,3)$.

Find the co-ordinates of the centre of gravity of the remaining lamina.



(a)

$$\bar{x} = \frac{3(-4)+7(2)+10(x)+15(-1)}{35}$$

$$x = 1.3 \qquad 10$$

$$\bar{y} = \frac{3(-5)+7(1)+10(y)+15(3)}{35}$$

$$y = -3.7 \qquad 10$$

(b)

	area :	c.g.	
pqr	$\frac{1}{2}(12)(6)=36$	$(4,2)$	5
psr	$\frac{1}{2}(12)(3)=18$	$(5,1)$	5
$pqsr$	$:18$	(x, y)	5

$$18(x) = 36(4)-18(5) \qquad 5$$
$$x = 3$$

$$18(y) = 36(2)-18(1) \qquad 5$$
$$y = 3$$

co-ords of c.g. $(3, 3)$ 5

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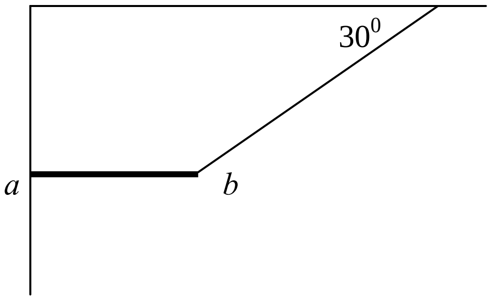
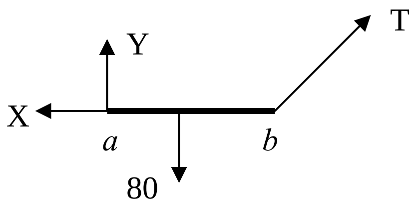
7. A uniform rod , *ab*, of length 4 m and weight 80 N is smoothly hinged at end *a* to a vertical wall.
One end of a light inelastic string is attached to *b* and the other end of the string is attached to a horizontal ceiling.
The string makes an angle of 30^0 with the ceiling, as shown in the diagram.
The rod lies horizontally and in equilibrium.



(i) Show on a diagram all the forces acting on the rod *ab*.
(ii) Write down the two equations that arise from resolving the forces horizontally and vertically.
(iii) Write down the equation that arises from taking moments about point *a*.
(iv) Find the tension in the string.
(v) Find the magnitude and direction of the reaction at the hinge.

(i)



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(ii) $X = \text{T}\cos 30$ 5

$Y + \text{T}\sin 30 = 80$ 5

(iii) $\text{T}\sin 30\left(4\right) = 80\left(2\right)$ 10

(iv) $\text{T}\sin 30\left(4\right) = 80\left(2\right)$

$$\text{T}\left(\frac{1}{2}\right)\left(4\right) = 160$$

$T = 80 \;\text{ N}$ 10

(v) $X = 80\cos 30 = 40\sqrt{3}$

$Y = 80 - 80\sin 30 = 40$

$\text{R} = \sqrt{\text{X}^2 + Y^2} = 80 \;\text{ N}$ 5

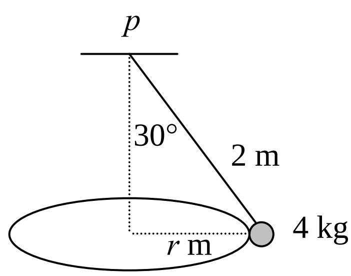
$\alpha = \tan^{-1}\left(\frac{Y}{X}\right) = \tan^{-1}\left(\frac{40}{40\sqrt{3}}\right) = 30°$ 5 50

8. **(a)** A particle describes a horizontal circle of radius 2 metres with constant angular velocity ω radians per second.
The particle completes one revolution every 5 seconds.

(i) Show that ω is equal to $\frac{2\pi}{5}$.

(ii) Find the speed and acceleration of the particle.
Give your answers correct to one place of decimals.

(b) A conical pendulum consists of a particle of mass 4 kg attached by a light inelastic string of length 2 metres to a fixed point p.



The particle describes a horizontal circle of radius r. The centre of the circle is vertically below p.
The string makes an angle of 30^0 with the vertical.

Find **(i)** the value of r
(ii) the tension in the string
(iii) the speed of the particle.

(a)

(i) $$\frac{2\pi}{\omega} = 5 \Rightarrow \omega = \frac{2\pi}{5}$$ 5

(ii) $$v = r\omega = 2\left(\frac{2\pi}{5}\right) = \frac{4\pi}{5} = 2.5 \text{ m/s}$$ 10

$$a = r\omega^2 = 2\left(\frac{2\pi}{5}\right)^2 = \frac{8\pi^2}{25} = 3.2 \text{ m/s}^2$$ 10

(b)

(i) $$r = 2\sin 30 = 1 \text{ m}$$ 5

(ii) $$T\cos 30 = 4g$$
$$\Rightarrow T = \frac{80}{\sqrt{3}} \text{ N}$$ 10

(iii) $$T\sin 30 = \frac{mv^2}{r}$$
$$\left(\frac{80}{\sqrt{3}}\right)\left(\frac{1}{2}\right) = \frac{4v^2}{1} \Rightarrow v = 2.4 \text{ m/s}$$ 10

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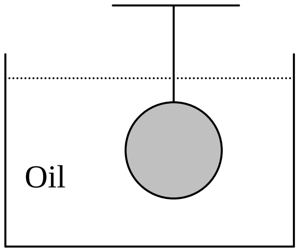
9. **(a)** State the Principle of Archimedes.

A solid piece of metal weighs 150 N in air and 131 N in water.
Find the volume of the piece of metal.

(b) A solid sphere of radius 5 cm and relative density 8 is completely immersed in oil of relative density 0.9.



The sphere is held at rest by a light inelastic vertical string which is tied to a fixed support.

Find the tension in the string.

[Density of water = 1000 kg/m^3].

(a)

(i) : Principle of Archimedes 5

(ii) $B = 150 - 131 = 19$ 5

$B = \rho V g$

$19 = 1000(V)(10)$ 5

$V = 0.0019 \text{ m}^3$ 5

(b)

$T + B = W$ 5

$T + \dfrac{W s_L}{s} = W$

$T + \dfrac{W(0.9)}{8} = W$

$T = \dfrac{71W}{80}$ 10

$= \dfrac{71}{80}\left\{8000\left(\dfrac{4}{3}\pi(0.05)^3\right)10\right\}$ 10

$T = 37.19 \text{ N}$ 5

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